1. INTRODUCTION

This work concerns the factorization of elliptic operators, namely the decomposition of a second order elliptic boundary value problem, defined in an open bounded regular domain, in a system of uncoupled first order initial value problems, using the technique of invariant embedding. The method presented here is a return, in a new spatial approach, to the technique of the invariant temporal embedding, defined originally in the context of Dynamic Programming and used in Control Theory for the computation of the optimal feedback.

2. FACTORIZATION BY INVARIANT EMBEDDING

The invariant embedding technique consists in embedding the initial problem in a family of similar problems depending on a parameter, which are solved recursively. In our approach, each problem is defined over a sub-domain limited by a mobile boundary (see Fig 1), depending on the parameter. Defining an operator relating the value of the solution, or its derivative, with the mobile boundary condition, we find a family of operators on functions of the section satisfying a Riccati equation and relating the boundary conditions on the section (Dirichlet-Neumann or Neumann-Dirichlet, for example).

For a given problem, this invariant embedding method is not unique: for instance we can apply the method either to the family of subdomains described above, either to the family of complementary subdomains; also, it is possible to change the type of condition that we impose over the mobile boundary. Without loss of generality, here we particularize the study to a Poisson equation with a Dirichlet boundary condition: \(-\Delta u = f\) in \(\Omega\), \(u|_{\Gamma_a} = 0\). We present the case where the family of curves which limits the subdomains defined by the invariant embedding are homothetic to one another and homothetic to a point, and we consider the moving boundary starting on the outside boundary of the domain and shrinking to a point. We show some results dealing with the singularity that will appear at that point.

Fig. 1. Invariant embedding in a star shaped domain.

REFERENCES
