Mathematical challenges and fast solution methods in aerodynamic shape optimization

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Keywords: Aerodynamic shape optimization, one-shot optimization, approximate reduced SQP

1. AERODYNAMIC SHAPE OPTIMIZATION

Aerodynamic shape optimization is a rewarding field for numerical analysis, since it poses several challenges due to the complexity of the subproblems involved and because efficient numerical solution methods have a high economical impact. This is particularly evident in the collaborative effort MEGADESIGN (2003-2007, funded by BMWi), whithin which most of the results presented here have been achieved. The focus of our research lies in the generation of fast numerical methods for optimal shapes of parts of the geometry of civil aircrafts. The objective for the optimization is the minimization of aerodynamic drag, because this immediately reduces fuel consumption. The aerodynamic models are represented in the form of complex flow simulators which are provided by our application partner. They involve costly iterative processes. The *main challenge* in aerodynamic shape optimization is to avoid wrapping another optimization loop around the simulation iterations, but rather to perform optimization steps already during iterative process of the forward simulation tool. Thus, this methodolgy is frequently called a "one-shot-optimization" method.

2. A NOVEL ALGORITHMIC TEMPLATE

From an abstract point of view, we consider optimization problems of the form

$$\min f(u,q) \tag{1}$$

$$\text{s.t. } c(u,q) = 0 \quad \exists c_u^{-1} \tag{2}$$

where u denotes the aerodynamic flow state variables, to be solved for in the flow equation. The

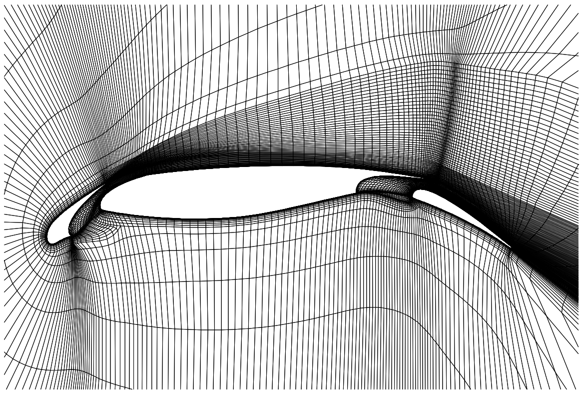

Fig. 1. Three part wing in high lift configuration

letter q denotes a finite dimensional parameterization of the shape that we investigate. The scalar valued function $f(u,q)$ denotes the objective of the optimization, i.e., here the aerodynamic drag resulting from the shape chosen. With this abstract problem formulation, we observe that the problem under investigation falls into the class *PDE constrained optimization problems*, which are of major interest in several international research programs.

By use of the Lagrangian function

$$\mathcal{L}(u,q,\lambda) = f(u,q) - \lambda^* c(u,q)$$

we can formulate our basic one-shot algorithm in the form

$$\begin{aligned}
\lambda^{k+1} &= \lambda^k - (A^*)^{-1}\nabla_u \mathcal{L}(u^k, q^k, \lambda^k) \\
q^{k+1} &= q^k - S_A^{-1}\nabla_q \mathcal{L}(u^k, q^k, \lambda^{k+1}) \\
u^{k+1} &= u^k - A^{-1} c(u^k, q^{k+1}, \lambda^{k+1})
\end{aligned}$$

where $A \approx \partial c/\partial u$ and S_A is an approximation to what we call the "consistent" reduced Hessian. This algorithm is highly modular but nevertheless iterates simulateously over all variables: state, design and adjoints.

3. MAJOR ISSUES OF THE TALK

The algorithm sketched above can be interpreted as an approximate variant of reduced SQP methods. By use of this interpretation, it is easily generalizable to completely different application areas outside of aerodynamic shape optimization with similar proble characteristics. These characteristics are that one wants to stick to an a-priorily implemented system solver and on the other hand has an adjoint solver at hand. At the core of the numerical analysis of this algorithm lie convergence results. These have been achieved so far for linear-quadratic model problems. We show the essential ideas of these convergence proofs. In addition to the problem formulation (1, 2), one has to treat state constraints modeling the lift requirement for the airplane and also a required pitching moment. Since these are scalar valued constraints, they can be included within the algorithmic framework by adding further adjoint equations, which are solved ”on the fly” as well. Enhancements of the convergence theory justifying this constraint treatment are presented, too. In a practical environment, practical issues of the implementation play a major role for the success of the resulting methods. These issues will be discussed together with several numerical results, e.g., for the three part wing depicted in Figure 1.

The success of the one-shot methods described within the talk has stimulated research towards further aspects: substitution of a fixed shape parameterization by a fine resolution based on the flow grid, incorporation of multigrid ideas in the algorithmic framework, the efficient treatment of uncertainties in the industrial environment, etc. Several of these aspects and recent research results for them will be discussed, as well.

4. CONCLUSIONS

A major part of the talk will be devoted to summarizing the research results presented in the publications (Gherman et al., 2005; Gherman, 2007; Hazra et al., 2004, 2006; Ito et al., 2006; Kroll et.al, 2004; Schulz, 1998, 2004) and so far unpublished results based on them.

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