SCHEDULING PROBLEMS FOR SYNCHRONIZATION OF MULTI-OBJECTS MOVEMENT

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1. INTRODUCTION

In the paper a problem of determining movement schedule of many objects is considered. The problem is used in many domains such as: routing in computer networks, movement planning of mobile robots, tasks processing in parallel or distributed computing systems, arms control of independent robots, planning and synchronization of many objects movement in computer simulation games, e.g. in Computer Generated Forces (CGF) systems (Petty, 1995). A special type of movement is such one that objects must be moved simultaneously. And a special type of system with this requirement is system for movement planning and simulation of military objects (units) in combat simulators. Movement scheduling has an influence on accuracy, adequateness, effectiveness and other characteristics of these systems. Then, the problem is to model and optimize such movement of detachments to achieve intended goals of commands (such as: achievement of destinations on restricted time, avoiding of losses during redeployment etc.). Regardless of kind of military actions military objects are moved according to some group pattern. For example, each object being moved in group (e.g. during attack, during redeployment) must keep distances between each other inside group. Therefore, the paper presents some problems of movement scheduling for many objects to synchronize their movement.

2. DEFINITIONS AND NOTATIONS

We assume that we have Berge’s graph G defining structure of the terrain (divided on the squares, hexagons, etc.) \( G = (V_G, A_G) \), \( V = |V_G| \), \( V_G \) – set of graph’s nodes (as centre of terrain squares), \( A_G \) – set of graph arcs, \( A_G \subseteq V_G \times V_G \), \( A = |A_G| \). We assume that on each arc we have defined value \( d_{e, d} \) of function \( d \) which describes terrain distance between the graph nodes \( n \) and \( n' \). We have \( K \) objects (columns, trucks, tasks) for movement from the vector \( s = (s_1, s_2, \ldots, s_K) \) of source nodes to the vector \( t = (t_1, t_2, \ldots, t_K) \) of destination nodes of \( G \). For further discussion we accept following notations:

\[
I_k(s_k, t_k) = I_k = \{i^0(k) = s_1, \ldots, i^r(k), \ldots, i^K(k) = t_k\}
\]

\[
T_k(I_k) = T_k = \{\bar{r}^0(k), \bar{r}^r(k), \ldots, \bar{r}^K(k) = \tau_k\}
\]

\[
V_k(I_k) = V_k = \{v^0_1(k), v^0_2(k), \ldots, v^K_1(k), v^K_2(k)\}
\]

where \( I_k \) - vector of nodes describing path for the \( k \)-th object, \( T_k \) - vector of time instances of achieving the nodes belonging to the path for the \( k \)-th object; \( V_k \) - vector of velocities \( v^0_1(k), v^0_2(k), \ldots, v^K_1(k), v^K_2(k)\) of the \( k \)-th object on the path for the \( k \)-th object; \( s_k, t_k \) – source and destination nodes for the \( k \)-th object; \( \bar{r}^r(k) \) - time instance of achieving node \( i^r(k) \) by the head of the \( k \)-th object, \( \tau_k \) - time of achieving destination node by the \( k \)-th object; \( V_k \) - vector of velocities \( v^0_1(k), v^0_2(k), \ldots, v^K_1(k), v^K_2(k)\) of the \( k \)-th object on the arc \( (i^r(k), i^{r+1}(k)) \) of its path; \( R_k \) - number of arcs belonging to the path of the \( k \)-th object.

Let \( IP_k = \{i^p_1(k), i^p_2(k), \ldots, i^p_{r_p}(k), \ldots, i^p_{R_p}(k)\} \) denotes set of nodes at which we must align the head of the \( k \)-th object in relation to the heads of other objects, where \( i^p_r(k) \) - the \( p \)-th element of \( IP_k \) satisfying:

\[
\forall r \in \{1, \ldots, R_k\}, \exists q \in \{1, \ldots, R_k\} \iff i^p_r(k) = i^p_q(k)
\]

\[
r^p_r(k) = r \in \{1, \ldots, R_k\} \iff i^p_r(k) = i^p_q(k).
\]

The form of \( IP_k \)
and $r_p(k)$ say that path for the $k$-th object must cross by nodes belonging to $IP_k$. Let, by analogy $TP_k = \{r_1(k), r_2(k), \ldots, r_{k-1}(k), r_k(k)\}$ denotes set of time instances of achievement particular alignment nodes from the set $IP_k$ by the $k$-th object head, $\tau_p(k)$ denotes moment of achieving the $p$-th alignment node by the $k$-th object,

$$\tau_p(k) = \tau^i(k) + \sum_{r=0}^{R_k-1} \frac{d(r^j(k))}{v(r^j(k))}$$

We make additionally assumption that $P_1 = P_2 = \ldots = P_K = N$, i.e. for all objects the same number of alignment points (nodes) exist. Moreover, we define for each $p = 1, \ldots, N$ following characteristic: $\tau^\max_p = \max_{k=1,\ldots,K} \tau_p(k)$.

### 3. FORMULATION OF THE PROBLEM

We define the problem of synchronous movement of $K$ objects as follows: for each $k \in \{1, \ldots, K\}$ to determine the path $I_k$ crossing by points from $IP_k$ and for each arc $(i^j(k), i^j+1(k))$, $r \in \{0, \ldots, R_k-1\}$ belonging to the path $I_k$ to determine such a velocity $0 < v(i^j(k), i^j+1(k)) \leq v^\max(k)$, that some goals (one or more) are satisfied, where $v^\max(k)$ describes maximal velocity of the $k$-th object resulting from its technical properties.

The most important goals for movement synchronization can be divided into two categories. The first category is time of movement of $K$ objects. We can define two basic measures of this category:

$$\tau^\max = \max_{k=1,\ldots,K} \tau^R_k(k), \quad \sum_{k=1}^{K} \tau^R_k(k)$$

The second category is “distance” between times of achieving alignment points by all of $K$ objects. We can define two main measures of this category:

$$\min_{p=1,\ldots,N} \left( \tau^\max_p - \tau_p(k) \right), \quad \min_{p=1,\ldots,N} \left( \sum_{k=1}^{K} \tau^\max_p - \tau_p(k) \right).$$

One of the formulations of optimization problem for movement synchronization of $K$ objects using defined two categories of measures can be presented as follows: for fixed paths $I_k$ of each $k$-th object to determine such $v(i^j(k), i^j+1(k))$ that

$$\sum_{p=1}^{N} \sum_{k=1}^{K} \tau^\max_p - \tau_p(k) \rightarrow \min$$

with the constraints:

$$v(i^j(k), i^j+1(k)) \leq v^\max(k), \quad r = 0, R_k-1, \ k = 1, \ldots, K$$

$$v(i^j(k), i^j+1(k)) > 0, \quad r = 0, R_k-1, \ k = 1, \ldots, K$$

Paths for $K$ objects may be disjoint or not and they must cross by fixed alignment points or we dynamically determine these points.

### 4. CONCLUSIONS

In the final version of the paper a nonlinear movement scheduling problem in order to minimize sum of delays of all ($K$) objects in checkpoints with some additional constraints will be defined. Two equivalent formulation of two-criteria mathematical programming problems taking into account two categories of the goals will be also presented. It will be proved that constraint coefficient matrices for both problems are totally unimodular and we can use effective algorithms for solving linear programming problems to find lexicographic solution of two-criteria problems. Two algorithms for synchronous movement scheduling will be proposed and their properties will be shown. Similarities and differences between defined problems and classical tasks scheduling will be proposed and their properties will be shown. Some extensions of moving scheduling problem will be presented, too.

### REFERENCES


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