COMPARISON OF THE EXACT AND APPROXIMATE ALGORITHMS IN THE RANDOM SHORTEST PATH PROBLEM

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1. INTRODUCTION

The determination of the shortest path in a given graph i.e. the classical shortest path problem (CSPP) is one of the basic problems of computational geometry. It has been discussed and many algorithms were proposed in the literature. See, for instance (Ahuja et.al, 1993), (Cormen et.al, 1990). Usually the authors consider the deterministic models where distances between vertices are deterministic. In only a few papers this problem was stated in random case when the distances are defined by random variables and some algorithms were proposed (Murthy et.al, 1996), (Murthy et.al, 1997).

In this paper the static random shortest path problem with the second moment criterion is discussed and a comparison of proposed exact and approximate algorithm is given.

2. PROBLEM STATEMENT

Let $G = (V, E)$ be a directed graph with a finite set of vertices V and a set of edges $E \subseteq V \times V$. Further, let $s \in V$ be a source vertice and let $t \in V$ be a destination vertice.

In the Random Shortest Path Problem (RSPP) each edge $e \in E$ is associated with a random variable T_e taking positive values. The goal is to find the path, from the source vertice s to the destination vertice t, which minimize a function of moments of random variables related with edges of the path. We also assume that the random variables T_e, for $e \in E$, are independent. We propose to consider a criterion that is a minimal sum of the variance and a square of the expected value of the sum of random variables related with edges of the path, i.e. the second moment of the sum of random variables related with edges of the path, because of $(\mathbf{E}[X])^2 + \mathbf{V}[X] = \mathbf{E}[X^2]$. Similar criterion has been considered by I. Murthy and S. Sarkar in (Murthy et.al, 1996).

Now we formulate the RSPP with criterion of the second moment. Let

$$\mathcal{P} = \{\langle v_0, v_1, \ldots, v_{n-1}, v_n\rangle | v_0 = s \wedge v_n = t \wedge \wedge (v_0, v_1), \ldots, (v_{n-1}, v_n) \in E\}$$

be a set of all paths from the source vertice s to the destination vertice t. The goal in the RSPP is to find a path $\langle v_0, v_1, \ldots, v_{n-1}, v_n\rangle \in \mathcal{P}$ such that

$$\mathbf{E}\Big[\Big(\sum_{i=1}^{n} T_{(v_{i-1},v_i)}\Big)^2\Big] = \min_{\langle u_0,u_1,\ldots,u_{m-1},u_m\rangle \in \mathcal{P}} \mathbf{E}\Big[\Big(\sum_{i=1}^{m} T_{(u_{i-1},u_i)}\Big)^2\Big].$$

To show that the problem can not be solved directly, for instance by Dijkstra algorithm it is presented an example of a random graph where the application of the Bellman Principle of optimality fails, i.e. where the subpath of the shortest path is not the shortest subpath.

3. MAIN RESULTS

In this section exact algorithms and approximate algorithms are introduced. It will be shown that the RSPP with the second moment criterion reduces to the Multi-objective Shortest Path Problem (MOSPP), thus the RSPP can be solved with a help of general methods for solving the MOSPP. First, two exact algorithms are presented, namely Extended Bellman-Ford algorithm (EBF) with a modified procedure of the relaxation of edge and a Generic Label Correcting algorithm (GLC) which is a kind of generalization of the classical Dijkstra algorithm. In this algorithm relaxations of edges are coming in a little bit more natural order than in the EBF algorithm. Unfortunately, the time complexity and also the space complexity of these general algorithms are exponential. Therefore a few approximate algorithms with the polynomial time complexity are proposed. One of them is a modification of EBF algorithm in which we have to assign with each vertice the list of all nondominated cost vectors related with all paths to this vertice. Unfortunately, for this reason EBF algorithm has an exponential complexity. The proposed approximate algorithm is a natural modification of EBF which assigns with the each vertice the list of a fixed number (the same for all vertices) of cost vectors. Another approximate algorithm is the Single Criterion Approximation algorithm (SCA). The idea of the algorithm is based on the fact that the RSPP with the second moment criterion reduces to the MOSPP. In this algorithm we permanently calculate the shortest path with respect to the one criterion and remove from the graph the edge with the greatest value of the second criterion.

The last proposed approximate algorithm is a variant of the algorithm presented in (Tsaggouris et.al, 2005) and (Tsaggouris et.al, 2006). The algorithm is a modification of classical Bellman-Ford algorithm to the MOSPP. In this algorithm we first estimate for each vertice v the minimal and the maximal distance from the source vertice s to the vertice v. Next, if some path from the source vertice s to the vertice v has the first criterion cost μ then we store the second criterion of the path at the position

$$\left\lceil k \cdot \frac{\mu - \mu_{min}(v)}{\mu_{max}(v) - \mu_{min}(v)} \right\rceil,$$

where μ_{min}, μ_{max} are the estimated minimal and maximal distances and k is the size of the array.

4. COMPUTATIONAL RESULTS

We performed our tests on quite large randomly generated graphs. Moreover, used graphs had from 10000 to 30000 vertices and each vertice had 10 outcoming edges. All the tests was made on Intel Celeron Mobile 1400MHz with 256MB RAM, working under control of Linux operating system with kernel version 2.6.11-6. Obtained results shows that the first proposed approximate algorithm works very well, i.e. its works very quickly and usually produce exact answer. The SCA algorithm is not so good and the last one works very longly, because the main loop of this algorithm usually can not be broken.

5. CONCLUSIONS

Computational results of the test of the presented exact and approximate algorithms, show that the approximate algorithms are much faster than the exact ones and could be very useful, for example, as a first approximation stage of some more difficult algorithm solving the RSPP. Moreover, carried out detailed analysis of different data representations can be used for solving the RSPP with criterion of higher moments.

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