ON EXISTENCE RESULTS FOR INFINITE HORIZON OPTIMAL CONTROL PROBLEMS

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1. INTRODUCTION

Still at the beginning of the previous century the optimal control problems with infinite horizon became very important with regards to applications in economics and biology, where an infinite horizon seems to be a very natural phenomenon, (5), (3), (10). Since then these problems were treated by many authors and various necessary, sufficient as well as transversality conditions were obtained, see for instance (6), (9). The question of existence of optimal solution was investigated among others by (1), (2), (4), (11) and (12).

2. THE MAIN PROBLEM CONSIDERED

The main problem we consider in this talk is formulated as follows. Minimize the functional

$$J(x,u) = \int_0^\infty r(t,x(t),u(t))\tilde{\nu}(t)dt$$

subject to all pairs

$$(x,u) \in\in W_p^{1,n}(\mathbb{R}^+,\nu) \times L_p^r(\mathbb{R}^+,\nu) \ ,$$

satisfying almost everywhere on $\mathbb{R}^+$
the state equations

$$\dot{x}(t) = f(t,x(t),u(t))\,,$$

the control restrictions

$$u(t) \in U, \quad U \in Comp\,(R^r) \setminus \{\,\emptyset\,\},$$

the initial conditions

$$x(0) = x_0.$$

The integral in the functional J is understood in Lebesgue sense. The remarkable on this statement is the choice of the weighted Sobolev- and weighted Lebesgue spaces as state and control spaces respectively. The idea of considering the state trajectories in weighted Sobolev spaces was firstly mentioned in (8). The functions ν and $\tilde{\nu}$ are assumed to be continuously differentiable, integrable over $\mathbb{R}^+$ having its values in $(0,1]$. We call such functions weights. These considerations give us the possibility to extend the admissible set and simultaneously to be sure that the adjoint variable belongs to a reflexive Banach space.

3. THE MAIN RESULT AND CONCLUSIONS

As the main result we formulate an existence theorem for the formulated problem for concrete classes of function f. The good imbedding properties of the weighted Sobolev space, convexity of the integrand function in control variable and some growth conditions which are assumed to be satisfied by the functions r and its derivative allow us to ensure the weak lower semicontinuity of the integral functional involved in the problem statement. This in turn is important in order to use the generalized Weierstrass theorem for the proof of the existence result. Verification of the weak compactness of the feasible set frames the second part of the proof.
Finally we formulate several examples, such as Resource Allocation Model, demonstrating the applicability of the theorem.

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