In the paper, we consider ODE driven optimal control problems with bang-bang type extremals. The specific nature of bang-bang controls causes some difficulties in optimality and stability analysis. Usual convexity arguments like (strong) Legendre-Clebsch condition fail to hold, and the control discontinuity has to be taken into account. In recent years, substantial results on second-order optimality conditions have been obtained in [8], also [6], [1], or [7]. The solution stability under parameter perturbation was investigated e.g. in [5] and [2], [3].

Optimality and stability conditions therein are:
(i) bang-bang regularity assumptions (finite number of switches, excluding e.g. endpoints),
(ii) strict bang-bang properties (nonvanishing time derivatives of switching functions at switching points e.g.),
(iii) assumption of simple switches (switch of no more than one control component at each time),
(iv) appropriate second-order conditions (positive definiteness of related quadratic forms e.g.).

Stability properties for the switching points localization had been obtained from the so-called deduced finite-dimensional problem using standard sensitivity results from nonlinear programming, or from a shooting type approach applied to the first-order system of conditions in Pontryagin’s maximum principle e.g. [2], [4]).

Consider the parametric problem
\[
(P_h) \quad \min k(x(T), h) \\
\text{s.t.} \quad \dot{x}(t) = f(x(t), h) + g(x(t), h) u(t) \quad (\forall) t, \\
x(0) = x_0(h), \quad h \in \mathbb{R} - \text{ parameter}, \\
|u_i(t)| \leq 1, \quad i = 1, \ldots, m, \quad (\forall) t
\]

Stability investigations have shown
1. the differentiability of switching points w.r.t. parameters under conditions (i), (ii) for linear state systems \((f = Ax, g = B)\), cf. [2],
2. differentiable behavior and local uniqueness of structure of extremals for semilinear systems \((f = f(x), g = B)\) under (i), (ii), (iv), cf. [4],
3. Lipschitz behavior (and possible lack of differentiability) for \((P)\) in case of simultaneous switches of two control components.

Up to our knowledge, the latter result is new. The proofs are based on certain backward shooting approach for characterizing broken extremals and make use of nonsmooth Implicit Function Theorems. For illustration, an example will be provided.

REFERENCES