Stability and Sensitivity Analysis for Optimal Control Problems with a First-Order State Constraint and Application to Continuation Methods

J. Frédéric Bonnans and Audrey Hermant

CMAP, Ecole Polytechnique, INRIA Futurs, Route de Saclay 91128 Palaiseau France
hermant@cmap.polytechnique.fr

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This talk deals with stability and sensitivity analysis for optimal control problems of an ordinary differential equation with a first-order state constraint. We consider the case when the Hamiltonian and the state constraint are regular. Malanowski (2) obtained Lipschitz continuity and directional differentiability of solutions in L^2, using generalized implicit functions theorems in infinite dimensional spaces, and without any assumptions on the structure of the trajectory. Malanowski and Maurer (3) proved that the solution and multipliers are C^1 with respect to the parameter by application of the implicit function theorem to the shooting mapping, when there are finitely many junction times and strict complementarity holds. Under those assumptions the structure of the perturbed solutions is stable.

The shooting algorithm, known to provide the solution of optimal control problems with a very high precision and low cost, requires in return a careful initialization of all parameters, as well as a knowledge a priori of the structure of the optimal trajectory (number and order of boundary arcs and touch points). In practice, the latter is not known and even though, it remains difficult to initialize the shooting parameters. A method to make up for this difficulty is to combine the shooting algorithm with an homotopy (or continuation) method. Starting from an "easier" problem (e.g. the problem without the state constraint), one solves a sequence of problems depending continuously from a parameter. The more information we have on the continuity/differentiability of solutions and shooting parameters with respect to the homotopy parameter, the easier it is to follow the homotopy path, for example using a predictor-corrector algorithm if the homotopy path is C^1. It is well known that for first-order state constraints, touch points (locally unique times when the constraint becomes active) are nonessential, i.e., strict complementarity never holds, and hence the structure of solutions is not stable. Among the different possibilities, a touch point can become inactive on the perturbed problem, remain a nonessential touch point, or it may give rise to a boundary arc.

Our main result is that, under natural hypotheses, these are the only three possibilities. We provide first-order expansions of the solution, multipliers and of all the shooting parameters. The main idea of the proof is to introduce touch points as boundary arcs of zero measure in the shooting formulation, and apply Robinson's strong regularity theory to a system of equalities and inequalities, whose Jacobian corresponds to the optimality conditions of the tangent linear quadratic problem involved in the no-gap second-order optimality conditions (see (1)).

We present an application of those results to an homotopy method, whose novelty is to handle automatically changes in the structure (apparition/disparition of a boundary arc). Preliminary numerical results are given.

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